

Search for the Dirac Monopole with 30-BeV Protons*

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A search was made at the Brookhaven alternating gradient synchrotron for magnetic monopoles produced either in collisions of 30-BeV protons with light nuclei, or produced by γ rays secondary to these protons in the Coulomb field of protons or of carbon nuclei. In runs using 5.7×10^{15} circulating protons, no monopole-like event was found. This implies an upper limit for production in proton-nucleon interactions of about 2×10^{-40} cm². Experimental limits are also derived for the photoproduction of pole pairs.

IT has been shown by Dirac¹ that the existence of a magnetic monopole would not be incompatible with quantum mechanics as we know it providing the magnetic charge g of the monopole is related in a particular way to the electronic charge e . It is necessary that g/e be an integral multiple of $\hbar c/2e^2$. Thus, the smallest magnetic pole strength allowed, other than zero, is $(137/2)e$ or approximately 3×10^{-8} emu. Dirac suggested that the existence of such an entity might have something to do with the quantization of charge and that magnetic monopoles, though hitherto unobserved, might be made in pairs in very energetic processes.

That suggestion has stimulated several previous experimental searches for a magnetic monopole. The first experiment to be reported was that of Malkus,² who devised a simple but elegant collector and detector of monopoles which might be incident on the earth's atmosphere as a component of the primary cosmic radiation, or which might be created in the atmosphere through the agency of the ordinary energetic primaries. Malkus was able to set an upper limit of 10^{-10} cm⁻² sec⁻¹ for the flux of such monopoles from either source. More recently, Bradner and Isbell³ have described experiments carried out at the Bevatron in which they looked, also without success, for magnetic monopoles which might have been made in processes initiated by 6-BeV protons.

As accelerators of considerably higher energy have become available, the question has been reopened. We report here the results of a search carried out in the flux of 30-BeV protons and of secondary γ rays of the Brookhaven alternating-gradient synchrotron (AGS). Concurrently, experiments with the same purpose but a somewhat different approach have been carried out at the AGS machine of CERN.^{4,5}

1. ASSUMED PROPERTIES OF THE MONOPOLE

It is generally assumed, for no more compelling reason than simplicity, that the Dirac monopole, if it exists, will bear the charge $(137/2)e$, not some multiple thereof. The design of our experiment reflects this prejudice, and in the discussions to follow, unless otherwise stated, the magnetic charge g is assigned the magnitude $(137/2)e$. The sensitivity to multiply charged monopoles is considered in Sec. 10. One assumes, of course, invoking charge conservation, that magnetic poles must be created in pairs and, conversely, that an isolated pole cannot vanish. Given the manifest scarcity, not to say absence, of monopoles in ordinary matter, this promises practically unlimited life to any monopole once it has been macroscopically separated from its partner in creation.

Very little can be said *a priori* about the mass of the hypothetical monopole. Dirac ventured only the speculation that it might be of the order of magnitude of a nucleon mass. In our experiment the heaviest monopole that could be produced in pairs in nucleon-nucleon collisions would have a rest mass of 2.9 BeV, that being half the energy available in the center-of-mass system for a 30-BeV proton incident on a nucleon in the primary target. (The mass limit would be higher, if coherent production in proton-nucleus collisions by way of long-range forces were to be considered.) A value of the rest mass that has a certain numerical appeal, but no more serious claim as far as we know, is 2.4 BeV, a mass which would endow a magnetic monopole of charge $(137/2)e$ with a classical radius equal to that of the electron. Because this figure lies comfortably within the upper part of the range accessible in our experiment, we shall frequently adopt it as an example in discussing mass-dependent aspects of the experiment. Apart from questions of cross section, one may say that the present experiment extends the searched range of rest masses by a factor 3 over that covered earlier by Bradner and Isbell.³ No accelerator experiment which has a negative result can wholly supersede an experiment like that of

^dEtudes Nucleaires de Saclay, Seine et Oise, 1961), Vol. 1, p. 155.

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¹ P. A. M. Dirac, Phys. Rev. **74**, 817 (1948).

² W. V. R. Malkus, Phys. Rev. **83**, 899 (1951).

³ H. Bradner and W. M. Isbell, Phys. Rev. **114**, 603 (1959).

⁴ M. Fidecaro, G. Finocchiaro, and G. Giacomelli, Nuovo Cimento, **22**, 657 (1961).

⁵ E. Amaldi, G. Baroni, H. Bradner, J. de Carvalho, L. Hoffman, A. Manfredini, and G. Vanderhaeghe, *Proceedings of the Aix-en-Provence International Conference on Elementary Particles* (Centre

Malkus,² for the cosmic radiation provides primary protons above any reasonable threshold.

Equally little guidance is available on questions of production cross section. Because of the very strong interaction of magnetic monopoles, one cannot simply recast the formulas for electromagnetic production of electrically charged particle-antiparticle pairs. We are not aware of any reliable estimate of a production cross section. In this situation, the experimenter can only try to establish limits as low as possible, hoping that an eventual theory may lend his negative results some significance. It seems likely that photoproduction will be the first process to yield to theoretical treatment; one has no basis at all for discussing other interactions. With this in mind, we have paid particular attention to establishing a limit on the external photoproduction of pole pairs by γ rays. The threshold is quite high for the mass range we are interested in, if the extra momentum has to be transferred to a nucleon. For example, it takes a 17-BeV photon to make a pair of 2.4-BeV mass poles in the field of a proton. For production in the field of a carbon nucleus by 17-BeV γ rays the mass limit is 5.6 BeV.

2. THE BEHAVIOR OF MAGNETIC MONOPOLES IN MATTER

Although the fundamental properties of the magnetic monopole, its mass, its spin and, above all, its existence, cannot be confidently predicted, once its existence as a stable entity with a certain magnetic charge is postulated, many features of its behavior in matter can be foreseen. Cole⁶ and Bauer⁷ have investigated theoretically the collision loss of monopoles moving through matter. Harish-Chandra,⁸ extending an investigation begun in Dirac's paper, showed that there is no bound state of the two-body system, magnetic-monopole and electron. Malkus² and Eliezer and Roy⁹ discussed the possible binding of a magnetic monopole to a nucleus and to an atom or molecule. We shall review here only those aspects of magnetic-monopole behavior which are relevant to our experiment.

A magnetic monopole of strength g moving at high speed through matter suffers enormous ionization loss. It ionizes much like a relativistic particle with an electric charge $68 e$, but its ionization loss, unlike that of an electrically charged particle, is substantially velocity-independent so long as the velocity is well above that of the atomic electrons. As a general rule, a fast monopole may be expected to lose about $8 \text{ BeV/g} \cdot \text{cm}^{-2}$; its specific ionization is about one-third that of a fission fragment. This extraordinary and distinctive property has been relied on as a means of identification in all experiments so far, including this one.

When a monopole has been slowed by collision loss until it no longer ionizes effectively, its interaction with the Coulomb field of nuclei provides a fairly large elastic-scattering cross section. For example, the cross section for large-angle scattering of a monopole of speed $2 \times 10^8 \text{ cm/sec}$ and mass 2.4 BeV by a nucleus with $Z=6$ is about 10^{-22} cm^2 . This mechanism would serve to moderate further the energy of the monopole. However, an even larger elastic cross section is to be expected, for velocities below 10^7 cm/sec , as a consequence of the diamagnetic repulsion between the magnetic monopole and the electrons of the atom core. For monopoles of a few electron-volts energy or less, the cross section for scattering by diamagnetic repulsion is of the order of atomic size. It is possible to analyze these processes in much detail, but for our immediate purpose it is enough to observe that an energetic monopole—starting, say, with 1 BeV kinetic energy—will *quite certainly* be reduced to thermal energy within a few tenths of a g/cm^2 of its starting point. Its subsequent fate concerns us also.

Between a diamagnetic atom or molecule and a magnetic monopole there is, as we have already said, a repulsive interaction. It is much larger than any effect we are accustomed to associate with diamagnetism. As long as the monopole is well outside the atom, the repulsive force is proportional to r^{-5} and the potential energy may rise to an electron volt or so at the atomic radius. At this distance of approach the perturbation of the electronic structure makes an accurate estimate difficult, but the order of magnitude can hardly be wrong. In a purely diamagnetic environment, then, we expect the monopole to remain free in the sense that it is not bound to any atom. Nevertheless, it may be bound within a lattice of atoms or ions in consequence of the repulsion which creates potential minima at interstitial positions. The situation is reminiscent of the so-called clathrate compounds in which a chemically inert atom is caged within a foreign crystal lattice. In a compact diamagnetic lattice of heavy atoms the monopole moves in an effective periodic potential which has minima a few tenths of a volt deep. Although this might appear adequate to trap monopoles at room temperature, the mobility of such trapped monopoles would be fairly high in even a weak magnetic field. A field of one gauss, for example, which is equivalent to a field of 20 kV/cm applied to an ordinary ion, may cause the monopole to drift through such a potential with a speed of many cm/sec at room temperature. The process is the analog of ionic conduction in crystals and the mobility predicted is, of course, exponentially sensitive to the barrier height assumed, so that any quantitative estimate is very uncertain. We conclude that one cannot confidently decide, *a priori*, whether a monopole in a diamagnetic lattice will or will not be effectively immobilized against the influence of a weak field. There is little doubt, on the other hand, that a field of the order of several kilogauss will lower the potential barriers enough to cause

⁶ H. J. D. Cole, Proc. Cambridge Phil. Soc. **47**, 196 (1951).

⁷ E. Bauer, Proc. Cambridge Phil. Soc. **47**, 777 (1951).

⁸ Harish-Chandra, Phys. Rev. **74**, 883 (1948); see also W. V. R. Malkus, his references 1 and 2.

⁹ C. J. Eliezer and S. K. Roy, Proc. Cambridge Phil. Soc. **68II**, 401 (1962).

rapid migration of a monopole which experiences only diamagnetic interactions.

However, most substances contain at least a few paramagnetic sites, and we need to examine the question of the binding of a magnetic monopole to a paramagnetic structure. The interaction can very roughly be estimated as follows. Consider an isolated atom containing an unpaired electron spin, and a monopole of charge g some distance r away. The magnitude of the Zeeman energy is $ehg/2m_e c r^2$. There is an attractive potential corresponding to a ground state in which the electron spin is polarized favorably along the atom-monopole axis. This description is valid providing the spin precession rate, in the field g/r^2 , is fast compared to the motion of the atom-monopole axis, a criterion which is satisfied with room to spare in the case of a massive monopole. If we now combine this r^{-2} attractive potential with the r^{-4} repulsive potential arising from the diamagnetic interaction of the monopole with the electron orbits, we obtain a potential well with a depth, typically, of a few volts, the minimum occurring at an r around 10^{-8} cm. The distance is small enough so that our simple representation of both the diamagnetic and the paramagnetic interaction can hardly be a good approximation. Still, the indication is strong that we may expect binding of the monopole to a paramagnetic atom or ion with a well depth measured in volts. This was also the conclusion of Malkus.²

A similar argument applied to the two-body system consisting of a nucleus or nucleon with a magnetic moment and a magnetic monopole indicates that there is no bound state. The question is a delicate one, however, for the margin by which a bound state is excluded is not vast, and one cannot be quite sure that a more refined analysis would not restore the possibility of binding in some cases. And, of course, there may be specific interactions between monopoles and nucleons of which we are not aware. We shall discuss later the implications for our experiment of a bound monopole-nucleus complex.

The question of the binding of stopped monopoles in matter can be avoided, up to a point, by stopping them in a fluid. If it is surrounded by a fluid, a monopole, whether it be bound to an atom, molecule, or molecular complex, must drift in the direction of an applied magnetic field until it comes to the boundary of the fluid. Its speed will be such that the viscous drag on whatever the monopole is bound to just balances the magnetic force gB . In a liquid of viscosity 1 cp, for example, the mobility of a structure of molecular size to which a monopole has attached itself would be of the order of magnitude of $10 \text{ cm sec}^{-1} \text{ G}^{-1}$. On the other hand, if the monopole remains unbound, it will likewise move in the direction of the magnetic force, the viscous drag being now provided by collisions of the bare monopole with atoms in its path. In a sufficiently weak field the "structure" which moves, in this case, may be described as a little bubble enclosing the mono-

pole, a bubble from which the liquid has been pushed by diamagnetic repulsion. The diameter of such a bubble can be estimated from the surface tension and bulk susceptibility of the liquid: It turns out to be, typically, of molecular size. Thus, in a weak field, the bound and the unbound monopole should move in much the same way through the liquid.

This hydrodynamic model can hardly apply if the driving field B is very strong. The energy dissipated along the monopole's track becomes so large that the local structure of the liquid may be altered. Or it may be that the force gB simply drags the bare monopole, if not the bound monopole, through the interstices of its molecular environment, causing it to surmount potential barriers and move with more than thermal velocity. A study of various models suggest that the division between weak and strong fields in the sense of this discussion lies around 100 G, as an order of magnitude. In any case we can be sure that the drift velocity of the monopole increases monotonically with increasing driving field, and that nothing can prevent its migration through the liquid in response to any magnetic field, however weak.

The inertial force involved in following a macroscopically curved line of magnetic force is relatively slight, as compared to the drag on the monopole in dense matter, under all practical circumstances. The extent of transverse diffusion can also be shown to be negligible. Hence, we may rely on the monopole following faithfully a field line in a homogeneous liquid.

Our experimental strategy was based in part on the preceding considerations, the idea being to avoid stopping the monopole in solid matter between creation and detection, to conduct it instead through a liquid by applying a magnetic field which would also serve to extract it from the surface of the liquid. The process of extraction from the liquid surface will be discussed in detail in Sec. 5 below.

3. OUTLINE OF THE EXPERIMENT

The experimental arrangement is shown in Fig. 1. A long straight section in the AGS machine is occupied by an evacuated target box which contains a flipping target shown at the left of the figure as the "primary target". This target consisted of a light material (Be, C, CH₂, Al) about 0.06 in. thick; it served a number of high-energy experiments with respect to which our experiment was a parasite. At the center of the straight section a well in the target box, 8-in. i.d., serves to intercept a fraction of such magnetic monopoles as might be created in a proton-nucleon interaction in the primary target and projected forward with a velocity comparable to that of the proton-nucleon center-of-mass system. Our "sample" monopole of mass 2.4 BeV would have a kinetic energy of 7.4 BeV at this velocity. The aluminum wall of the well is too thin (0.060 in.) to stop such energetic monopoles, at least if their charge

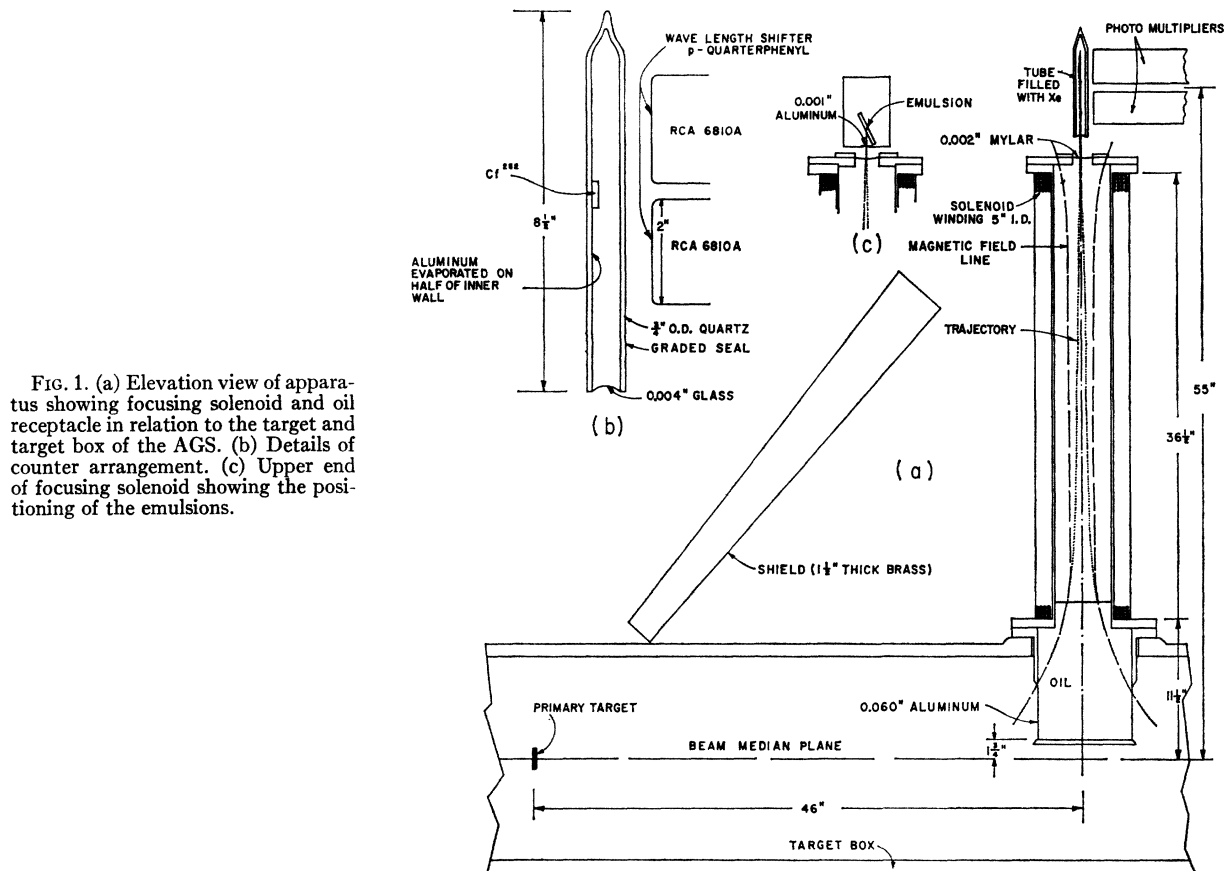


FIG. 1. (a) Elevation view of apparatus showing focusing solenoid and oil receptacle in relation to the target and target box of the AGS. (b) Details of counter arrangement. (c) Upper end of focusing solenoid showing the positioning of the emulsions.

is no greater than $(137/2)e$. They would penetrate it and be stopped within the liquid which fills the well. The liquid was Welch Duo-Seal Pump Oil.

The liquid has another function. It serves as a target for possible electromagnetic production of monopole pairs by the energetic photons which traverse the well. The oil "converter" is about half a radiation length thick and is favorably located to intercept a considerable fraction of the high-energy gamma-ray flux from the primary target.¹⁰

Mounted vertically above the well is a long solenoid by means of which a monopole can be accelerated to high energy for detection. The interior of the solenoid is maintained at a pressure around 100μ by a fore-pump, and is closed at the top by a 0.002-in.-thick Mylar window. A monopole of the appropriate sign, stopping in the oil, is drawn to the free surface of the oil by the field from the end of the solenoid. Assuming for the moment that it is there extracted from the oil as a bare monopole, it is accelerated in the evacuated region, arriving at the top of the solenoid with a kinetic energy near 1.1 BeV, a figure which depends, of course, only

¹⁰ On the downstream side the well was lined by $\frac{1}{4}$ in. of Pb. It is, however, uncertain (see Sec. 2) whether monopoles created in this material would be extracted by the rather weak magnetic field.

on the monopole charge, the field in the solenoid which was ordinarily 500 to 700 G, and the effective length of the solenoid, approximately 90 cm. The nonuniform field near the lower end of the solenoid was exploited, as explained in more detail below, to focus all monopole trajectories into a small aperture at the detector end. After passing through the Mylar window and a few centimeters of air the monopole enters the detector.

Two methods of detection were used: (I) a xenon scintillator consisting of a quartz tube filled with pure xenon and viewed by two photomultipliers, and (II) nuclear emulsions. Both detection methods relied on the high specific ionization of the magnetic monopole to distinguish it from the copious background of relativistic charged particles. This background could have been ignored in using detector (I) if we had been quite certain that the mobility of the monopoles in oil would not greatly exceed that estimated above for the weak-field case. According to that estimate, the arrival of a monopole at the detector should be delayed until well after the spill-out of the proton beam, and the counters could simply have been gated off during the spill-out period. It had been our original plan to operate the scintillation detector in this fashion, despite lingering uncertainty about the mobility. Fortunately, it proved possible to cope with the background without gating the counter

so that, in the end, our conclusions do not depend on the assumption of a minimum delay.

The photomultipliers which viewed the xenon tube were connected to a fast coincidence circuit, discriminator, and fast oscilloscope. The discriminator level could be set with reference to pulses from fission fragments provided by an internal Cf^{252} source. Allowing for loss in the windows, a monopole would have deposited in the xenon nearly ten times the energy of a fission fragment. A discriminator setting around three times fission-fragment pulse height adequately suppressed the background; any pulses above this could be examined in detail on the oscilloscope photograph. For emulsion runs the xenon counter assembly was removed and replaced by a box with an aluminum window 0.001 in. thick, as indicated in Fig. 1(c).

In the early runs the solenoid was pulsed (to 760 G) to avoid any harmful influence of its stray field on the proton orbits at injection. The solenoid was switched on for approximately half the machine repetition period, from 0.2 sec before to 1.2 sec after beam spill-out. After it was ascertained that the stray field caused no trouble at injection, we ran with the solenoid on continuously with, however, its field reduced from 760 to 500 G to avoid overheating.

The total exposure in any run was most directly recorded in terms of the number of protons in the circulating beam summed over all machine pulses. In the whole experiment we accumulated a total of 5.7×10^{15} circulating protons without recording any monopole-like events. To establish the significance of this negative result and to dispose of a number of possible loopholes, we shall have to discuss in detail the critical features of the experiment.

4. FOCUSING

The field of the solenoid has a first-order focusing effect on the trajectories of monopoles which start from rest near one end of, and are accelerated in, the evacuated column. Its action is like that of an electrostatic immersion lens for ions. The trajectories are independent of field strength and monopole mass in the nonrelativistic approximation. The focal distance depends on the position, relative to the end of the solenoid, of the source plane which is here the liquid surface. The liquid level was set so as to bring the calculated focal point just above the upper end of the solenoid. A relativistic calculation of the focal length was actually used, although the shift of focal point with mass over the mass range of interest is not serious because the bundle of trajectories from the source fills a very small angle at the focus. This is seen in Fig. 1 which shows the trajectory of a monopole which has migrated to the surface from the upper edge of the thin section of the well wall. Monopoles originating below this would follow trajectories even closer to the axis.

It was necessary to make sure that magnetic fields from other sources would not deflect the monopole tra-

jectories away from the entrance to the detector. A survey of the ambient static field and of the stray field from the pulsed AGS magnets showed no horizontal component greater than 1 G. The location of the solenoid was especially favorable in this respect. It was midway between two AGS magnets of opposite orientation, so that the axis of the solenoid was at the same time an axis of symmetry of the nearby magnetic structure of the synchrotron. A uniform horizontal component of 1 G over the length of the solenoid would shift the focal spot by only 2 mm.

5. EXTRACTION FROM THE LIQUID

Suppose a monopole arrives at the surface of the oil bound to a molecule which it has dragged along. We have seen that binding to paramagnetic molecules is likely, and the presence of free radicals in the oil, especially oil exposed to intense ionizing radiation, can be taken for granted.¹¹ Will the molecule bearing the monopole evaporate? Evaporation is, of course, favored by the upward force gB , equivalent, in our field of 500 G, to a field of 10^7 V/cm acting on an ion. Now even without such encouragement, if the equilibrium vapor pressure over the oil is 1μ , the lifetime of a surface molecule against evaporation is of the order of 0.01 sec. If, therefore, the bonds between this particular molecule and its neighbors are not drastically strengthened by the presence of the monopole, the molecule will certainly evaporate, together with its monopole, becoming then a subject for the considerations of the following section.

One cannot, however, exclude the possibility that the monopole will associate around itself a complex of molecules. Indeed, the paramagnetic bond we have described has a nonsaturable character. It is conceivable that the monopole might collect a shell of molecules around it, limited in number only by steric effects. Moreover, because the paramagnetic attraction is a long-range force, and also because we can say so little about the chemical properties of a molecule whose electronic configuration has been altered by the presence of a magnetic monopole, we cannot flatly rule out the coagulation of a cluster even larger than one shell of molecules. If the field B is strong enough, however, escape of a heavily encumbered monopole can be guaranteed by the following essentially macroscopic argument.

Suppose that the influence of the monopole on the local mechanical properties of the liquid becomes negligible at some distance r_0 from the monopole. Make the extreme assumption that the region $r < r_0$ is bound, cross linked, or otherwise congealed into a solid ball—a ball on which, of course, the force gB still acts. Now the

¹¹ Paramagnetic resonance measurements, performed by Dr. A. J. Tench and Dr. N. Sutin, indicated a concentration of 10^{12} – 10^{13} spins/cm³ both in fresh and in previously irradiated oil. For this range of concentrations we are unable to estimate whether or not a pole on its path through the oil will become bound to a paramagnetic site. Spin concentrations *during* irradiation might, of course, be much higher.

maximum force required to extract a sphere of radius r_0 from the liquid is $2\pi r_0 T$, where T is the surface tension of the liquid, by assumption normal outside the sphere. The ball will then necessarily emerge if gB exceeds $2\pi r_0 T$. Take $r_0 = 10^{-6}$ cm. On this scale, a macroscopic picture of the deformation of the liquid surface is admissible. And surely, the influence of the monopole on the mechanical properties of the liquid cannot actually extend as far as this, for the field of the monopole at such a distance is only 30 kG. If we now put in 50 dyn/cm for T , we find that a field B of 10 kG will meet our criterion. Any relaxation of the assumed rigidity within $r = r_0$ can only make extraction easier.

We consider it unlikely that monopoles could have been prevented from leaving the surface of the oil in the 500-G field that was used in most of the runs. Nevertheless, it seemed worthwhile to make one run in which this remote possibility could be totally excluded. A small solenoid was mounted axially inside the large solenoid, just above the surface of the oil. This solenoid could be switched on for 2 sec to produce a field of about 10 000 G. After exposure of the oil target, with the emulsion detector put in place and with the large solenoid left on, the oil level was raised to a suitable height within the small solenoid, and the pulse of intense field was applied. Any monopoles that had accumulated at the surface would have previously migrated to the magnetic axis and, according to the argument above, must unquestionably have been extracted.

Two K0 emulsions, 400 μ thick, so exposed using a combined total of 1.0×10^{16} circulating protons, were scanned and no monopole-like track was found.

6. STRIPPING

Once the monopole-molecule complex has left the liquid, its acceleration by the magnetic force brings it to a velocity so great that a subsequent collision with a molecule of the vapor will disrupt the binding and strip the monopole from its encumbrance. To show that this will happen we note that the energy acquired from a field of 500 G in one molecular free path at 100- μ pressure is of the order of magnitude of 3 MeV. Here we have assumed a collision cross section of 10^{-15} cm². Even if the monopole shared this energy with a complex of molecular weight 10^3 , its relative velocity in a collisions would be high enough to disrupt any paramagnetic binding.

If the stripping collision were to occur only after the monopole-bearing molecule had traversed most of the length of the solenoid, the energy with which the monopole itself would arrive at the detector would, of course, be much less than we counted on. Thus, a very low pressure in the column is not desirable.

If one postulates attachment to a very large cluster of molecules, like that represented as a solid ball in the discussion of the preceding section, one can estimate the rate at which energy would be deposited in this quasi-

macroscopic ball as it is dragged through the gas. Also, the drag can be taken into account explicitly in the equation of motion. The indicated energy transfer to the ball is so large, under the prevailing conditions, that it would disintegrate in much less than 1 cm of travel, even if it started with a radius as large as 10^{-6} cm. We conclude that even under the most far-fetched assumptions of chemical binding, the monopole will be stripped by gas collisions near the bottom of the solenoid. The same argument applies should a monopole on its path through the oil (or through Al, see Sec. 10) have become paramagnetically bound to a number of oxygen molecules.

A different situation would be presented if the monopole were bound to a nucleus. Presumably such a structure would have dimensions in the nuclear range and a binding measured in MeV. In that case only a nuclear collision, highly improbable under the conditions of the experiment, would detach the monopole from its partner. The detector would be traversed by the compound object. The ionization loss would be dominated by the magnetic monopole charge, rather than the nuclear charge, and would be largely velocity independent. Therefore, the range and ionization should be very much the same, in either detector, as expected for the bare monopole accelerated through the same field.

A hypothetical structure more difficult to handle would be a monopole bound to a nucleus considerably heavier than itself in a structure of dimensions intermediate between atomic and nuclear size.¹² Stripping might then occur in the windows, or in the emulsion, and not in the accelerating column. The monopole, with only a fraction of the expected energy, would have lost its partner before detection. The partner would also traverse the scintillator or the emulsion, ionizing fairly heavily because its electrons would also have been pretty well stripped away, but not so heavily as to deposit all its energy within the range anticipated for the full-energy monopole. It seems likely that such an event would have escaped detection in both the scintillator and the emulsion.

7. FIRST DETECTION SYSTEM; XENON COUNTER AND ASSOCIATED ELECTRONICS

The most important feature required of a detector of monopoles in this experiment is the capability of indicating a rare, extremely heavily-ionizing particle in the presence of a copious background of relativistic charged particles and the gamma rays close to the AGS target. We have constructed a xenon gas scintillation counter which meets such a requirement.

It is well established that the light response of the rare-gas counter has a fast decay time (10^{-8} sec) and

¹² For a nuclear binding at any distance to occur, the monopole must first traverse the diamagnetic barrier of the electronic shells. This requirement makes the prevalence of such structures unlikely.

that its pulse height is a linear function of energy loss.¹³ The latter characteristic is desirable when detecting monopoles by pulse-height discrimination since their ionization density is between those of α particles and fission fragments. The narrow time spread of the light pulse is helpful in reducing the effect of pile-up of background pulses during the beam spill-out period.

Another advantage of the gas counter over a possible alternative, e.g., a solid-state detector, is that the sensitive volume of the counter can easily be given a shape which enhances the signal to background ratio in our particular experimental arrangement. That is, taking advantage of the focused trajectories of the monopoles, the longest dimension of the counter can be matched to the estimated range of accelerated monopoles (roughly 100 mg/cm²), while the lateral dimension can be much smaller. This ensures that most of the background particles have shorter tracks in the counter than monopoles do.

The scintillating volume of our counter was a transparent tube 8½ in. long, ¾-in. o.d., and 1.16-in. wall thickness, filled with purified xenon gas of spectroscopic grade at atmospheric pressure. The central part of the tube, 4½ in. long, was made of quartz in order to transmit scintillation light from xenon which is predominantly ultraviolet. One side of the inner wall of the quartz tube was covered with a layer of evaporated aluminum to increase light collection efficiency. Both end sections of the tube were made of Pyrex glass, joined to the quartz tube through graded seals. The lower end of the tube was sealed with a thin glass window of 4-mil thickness which corresponds to about 25% of the estimated range of monopoles of 1-BeV kinetic energy.

The xenon tube was vertically mounted on a rectangular bakelite frame which in turn was held against the faces of two RCA 6810A photomultipliers. In order to shift the spectrum of primary radiation to the sensitive region of the photocathode, a layer of *p*-quaterphenyl, approximately 40 µg/cm², was evaporated onto the surface of each phototube. In the space between xenon tube and photomultipliers, we deliberately employed an air gap to avoid any possible source of background due to Čerenkov radiation.

The whole counter assembly was carefully shielded against magnetic fields with a combination of Co-netic and Netic materials, enclosed in a rectangular steel box, and placed on top of the accelerating solenoid.

Pulses from the anodes of both photomultipliers were fed into a double-coincidence circuit via attenuators. The output of the coincidence circuit was recorded on a scaler and also used to trigger the sweep of a Tektronix 517 oscilloscope; the vertical input signal consisted of the two dynode pulses of both photomultipliers with time delays appropriate for a sequential display. The

time display covered an interval of 200 nsec. By adjusting attenuators on the anode pulses, we set a discrimination level for the coincidence. For each event which satisfied the coincidence condition, a picture was taken of the oscilloscope screen with a Polaroid camera. Thus, each event could be studied at leisure, pulse heights measured, and pulse shapes and time correlations examined.

In view of the uncertainty on the mobility of monopoles as discussed in Sec. 2, it seemed to be desirable to know the time relationship between the beam spill-out and the detection of an event. This was accomplished with an accuracy of 1 msec by arranging for the coincidence output to turn off a scaler counting a 1-kc signal, the latter signal commencing at zero time of the AGS acceleration period. Beam spill-out occurred approximately one second after zero time and had a duration of 15 msec. The precise spill-out time was monitored with an auxiliary counter-telescope and proved to have negligible time jitter with respect to zero time.

The xenon detector system was calibrated using a Cf²⁵² spontaneous-fission source.¹⁴ The source was evaporated on a thin circular platinum foil and placed along the inner wall of the xenon tube, equidistant from the two photomultiplier tubes. [See Fig. 1(b)]. In order to obtain a fission spectrum with good resolution, the dynode pulses of both photomultipliers were added, stretched, and fed into a pulse-height analyzer. The double-peaked fission spectrum as well as the low-energy alpha peak were observed. According to the time-of-flight measurements of Milton and Fraser,¹⁵ these peaks correspond to 104.7, 79.8, and 6.11 MeV, respectively. The height of the alpha peak was greatly reduced in our spectrum because of the limited solid angle in which alpha particles can travel their full range in the xenon gas. Nevertheless, the observed spectrum confirmed the linearity of our detector with energy loss over a wide range of specific ionization (about 30:1).

The monopoles, after being accelerated to approximately 1 BeV by the solenoid field, will deposit approximately 10 times as much energy in the xenon counter as the maximum-energy fission fragment. Thus, the built-in fission source provided a convenient reference for setting the discrimination level. In view of some uncertainty in the estimated range of the monopole, we normally set the bias at 2.8 times that of the maximum fission pulse. During a long run, we frequently lowered the bias and counted the upper part of the fission spectrum as a general check of the whole detection system. Pictures of the fission pulses were regularly taken in order to monitor any drift in photomultiplier gain.

There were two possible sources of spurious events which might give pulses large enough to trigger the coincidence circuit: (1) pile-up of small pulses due to the beam bunching, and (2) the production of stars with

¹³ A. Sayres and C. S. Wu, *Rev. Sci. Instr.* **28**, 758 (1957); J. A. Northrop, J. M. Gursky, and A. E. Johnsrud, *Proceedings of Sixth Scintillation Symposium* [I.R.E. Trans. Nucl. Sci. NS-5, 81 (1958)].

¹⁴ The source was kindly prepared for us by Dr. T. D. Thomas of Brookhaven National Laboratory.

¹⁵ J. C. D. Milton and J. S. Fraser, *Phys. Rev.* **111**, 877 (1958).

heavily-ionizing prongs. To avoid the former, we made provision to gate off our counter during the beam spill-out period by pulsing the voltage on the focusing grids of the photomultipliers. However, it turned out that pile-up did not produce pulses larger than twice the maximum fission pulse. Since a relativistic particle could not lose more than 0.2 MeV by ionization in passing through the xenon, it required well over 1000 particles bunched together within 20 to 30 nsec to produce a monopole-like pulse. Considering the geometry of our detector, the beam intensity (2×10^{10} protons per pulse), and the observed internal structure of the beam, this was a highly improbable event. Neither did we obtain coincidence events due to star production. Large pulses which might have been due to this process were observed only when the oscilloscope was triggered by either one of the two photomultipliers alone. The resultant photograph showed a large pulse in one counter and none at all in the other. This was suggestive of star production in the wavelength shifter or in the photocathode material itself.

8. SECOND DETECTION SYSTEM: EMULSIONS

In a second series of runs, nuclear emulsions were used as detectors. To make long exposure times possible, an emulsion of low sensitivity, namely Ilford K-minus-2, was selected. The plates were 1 in. \times 3 in. in area, 200 μ thick. The sensitivity was gauged in two ways. First, plates were exposed to a Cf²⁵² source. With the development chosen,¹⁶ the fission fragments produced heavy black tracks, and residues of α -particle tracks, consisting of arrays of 3-4 grains or of closely spaced pairs of grains, were visible. Next, an exposure to oxygen ions, of about 10 MeV/nucleon energy, was obtained. The ions, incident at an angle of 20° to the emulsion plane, produced tracks about 130 μ long, moderately light for the major portion of their range, but black or nearly black in the region of maximum ionization near the end of their range. In this region, oxygen ions lose about 1.5 MeV/ μ ,¹⁷ about two-thirds of the energy loss of monopoles of charge (137/2) e . As the ionization of monopoles is so large and nearly independent of range, we could thus feel confident that these particles would produce easily visible tracks.

For the monopole exposures, the plates were mounted in boxes and placed at an angle of 20° to the solenoid axis [see Fig. 1(c)]; they were centered within 0.5 mm. Fifty plates were exposed and each was left in position until 1×10^{14} protons had circulated in the AGS. This exposure produced a moderately heavy background of random grains, in which occasional light tracks (possibly due to light ions) and short ($\leq 10\mu$) heavier tracks (of heavier ions) could be found.

Because of the rapid fading of K-minus-2 emulsions (the above-mentioned residues of α tracks disappear

after 24 h of storage at room temperature), the plates were developed within 12 h, or less, after exposure. If the delay between exposure and development exceeded 3 h, the plates were stored at about 0°C, at which temperature the rate of fading is reduced.

An area corresponding to 1 cm² of beam, centered on the solenoid axis, was scanned for monopole tracks entering the emulsion surface. Such tracks should be about 350 μ long for charge (137/2) e , and half as long for twice that charge. In preparation, the scanners studied the oxygen and other ion tracks in order to find an appropriate scanning speed. The magnification was 500 \times ; the sample tracks could easily be seen with half this magnification.

No tracks that could be attributed to monopoles were found.

9. CONVERSION OF DATA INTO CROSS-SECTION LIMITS

As previously stated, we express our negative result in terms of upper cross-section limits for production of monopoles (a) by nuclear interactions and (b) by γ rays. An outline of the procedures used to arrive at these numbers follows.

(a) We assume that all pole pairs are produced in the primary target by the circulating protons and we neglect any production in the oil by scattered or secondary particles (compare Fig. 1). We assume further that all monopoles with the correct polarity (south) which enter the oil will there be brought nearly to rest, extracted from the oil, focused, and accelerated in the manner outlined in Secs. 4-6. For monopoles with characteristics as defined in the introduction, this assumption is quite valid. What few doubts remain are discussed in Sec. 10. Finally, we assume that any south monopole entering either detecting system would have been observed.

The number N_m of entering poles is related to the production cross section in nucleon-nucleon interactions σ_n by the equation

$$N_m = f N_p t N \sigma_n, \quad (1)$$

where N_p = number of circulating protons, t = target thickness in g/cm² \times average number of target traversals per proton, N = number of nucleons per gram of target = 6.0×10^{23} , and f = geometrical factor representing the fraction of monopoles produced which enter the oil.

To calculate an upper limit for σ_n , we used $N_m = 2$. Thus, for σ_n equal to the limit, the probability is 86% that one or more poles would have been observed. N_p was read on the AGS beam monitor which has an estimated accuracy of $\pm 10\%$.¹⁸ A conservative estimate of t is 20 g/cm².¹⁹ The geometric factor f was estimated by

¹⁸ H. J. Halama, Brookhaven National Laboratory Accelerator Development Report HJH-1 (unpublished).

¹⁹ For light element, t , as determined by radiochemical methods, is usually about 30-35 g/cm², but infrequently numbers as low as 10 g/cm² have been observed. We are indebted to Dr. G. Friedlander for providing us with this information.

¹⁶ 10g/l AgBr was added to the hypo to eliminate corrosion.

¹⁷ H. H. Heckman, B. L. Perkins, W. G. Simon, F. M. Smith, and W. H. Barkas, Phys. Rev. **117**, 544 (1960).

transferring the laboratory angles subtended by the oil vessel into the center-of-mass system of two colliding nucleons. Isotropic production in the latter system was assumed and the fraction of the total solid angle represented by the transformed laboratory solid angles was calculated for various monopole c.m. kinetic energies. For monopole masses m in the upper part of the interval accessible (in proton-nucleon collisions) in our experiment, say $2 \text{ BeV} < mc^2 < 2.9 \text{ BeV}$, typical values of the kinetic energy range from about $0.05 mc^2$ to $0.2 mc^2$, and an average value of f is 0.2.²⁰ The comparatively large fraction of intercepted monopoles is due to the high velocity of the center-of-mass system which concentrated the particles into a small cone in the laboratory system, together with the fact that the oil vessel intercepted nearly a quarter of all particles emitted from the target at angles between 2.5° and 8° . Excluding the exposure with a pulsed magnetic field (see Sec. 5), the total number of circulating protons included in our counter and emulsion runs was 5.7×10^{15} . Thus, according to (1), with $f=0.2$,

$$\begin{aligned} \sigma_{n, \text{max}} &= 2 / (0.2 \times 5.7 \times 10^{15} \times 20 \times 6.0 \times 10^{23}) \\ &= 1.4 \times 10^{-40} \text{ cm}^2 / \text{nucleon}. \end{aligned}$$

For lower monopole masses, $1 \text{ BeV} < mc^2 < 2 \text{ BeV}$, typical kinetic energies are higher, the cone of particles is wider, and f is smaller, roughly $f=0.1$. Thus, for these masses, $\sigma_{n, \text{max}} \approx 3 \times 10^{-40} \text{ cm}^2 / \text{nucleon}$.

The possibility might be considered that monopoles were created in the primary target, but were reabsorbed by the target nucleus in a secondary collision, due to some unknown strong interaction. However, as mentioned before, the target was polyethylene in part of the runs; it is estimated that in about 5% of the collisions the primary protons interacted with the hydrogen contained in this substance. Thus, if nuclei (other than protons) were ineffective in producing monopoles, the values of $\sigma_{n, \text{max}}$ would have to be increased by a factor 20.

(b) The apparatus was designed to be effective in detecting monopoles produced in the oil by γ rays from the primary target. In estimating upper limits for the cross section of this process, it was again assumed that any south monopoles produced in the oil would have been detected.

The estimates were based on the γ -ray spectra measured by Fidecaro *et al.*²¹ at 23.1 and 24.5 BeV proton energy and 3° , 3.2° , and 6° emission angles. As in about one-half of our runs the target was Be, C, or CH_2 , and in the other half Al, we used the average of Fidecaro's Be and Al data. The curves were extrapolated to somewhat higher energies by eye. (An extrapolation by

²⁰ The requirement that monopoles be emitted at an angle $> 2.5^\circ$ reduces the maximum obtainable mass by only 0.04 BeV.

²¹ M. Fidecaro, G. Gatti, G. Giacomelli, W. A. Love, W. C. Middelkoop, and T. Yamagata, *Nuovo Cimento* **19**, 382 (1961); M. Fidecaro, G. Finocchiaro, G. Gatti, G. Giacomelli, W. C. Middelkoop, and T. Yamagata, *ibid.* **24**, 73 (1962).

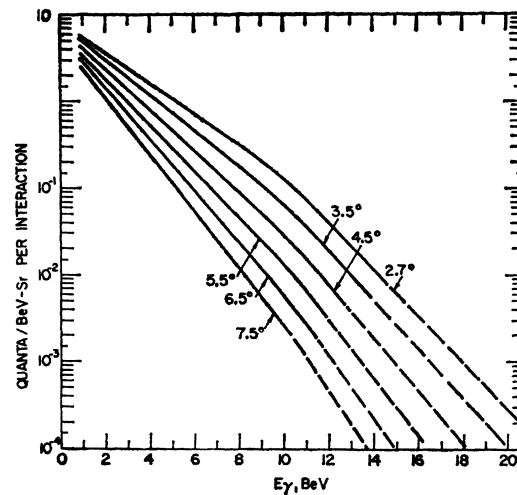


FIG. 2. Differential photon spectrum for 30-BeV protons incident on primary target for various emission angles. The solid curves are derived by a Lorentz transformation from experimental values obtained by Fidecaro *et al.* with 24-BeV protons. The dashed portions are extrapolations.

phase-space data was deemed to be even less reliable, as at the highest measured points the experimental curves appear to depart from the phase-space curves not only in intensity but also in shape.) On the assumption that the center-of-mass spectrum is independent of primary energy, a 30-BeV spectrum was simply obtained by a Lorentz transformation. (Due to this assumption the intensity of the upper part of the spectrum will be somewhat underestimated.) A small correction for the increase of π^0 multiplicity with proton energy was applied, but is actually negligible compared to other uncertainties. Spectra for other angles were obtained by interpolation. Resulting 30-BeV spectra are shown in Fig. 2 where the dashed portion of the curves corresponds to the above-mentioned extrapolation. At the highest energy shown the extrapolation may be in error by as much as a factor 10.

These spectra were then used to obtain the distribution of photon path lengths in oil for our experiment. To do this the oil vessel was divided into segments as seen from the target and a numerical integration performed to obtain the product photon number per interaction times path-length in oil, in cm, for each 1-BeV photon-energy interval from 2 to 20 BeV. These photon-number times path-length products were corrected for γ -ray absorption in the oil assuming a radiation length of 50 cm, and were multiplied by 2×10^{15} , the total number of interactions in the target corresponding to the effective target thickness $t=20 \text{ g/cm}^2$. The differential path-length spectrum so obtained was then converted to the integral spectrum $L(E_\gamma)$, the total path length in oil of quanta having an energy $> E_\gamma$. Both the integral and the differential path-length spectrum are shown in Fig. 3. The dashed portions of the curves depend on the extrapolations of Fig. 2. Be-

low 13 to 14 BeV, however, the uncertainty of the extrapolation affects the curves of Fig. 3 but little, as most of the photon path length is contributed at emission angles for which extrapolation was not involved.

On the fiction that all quanta above the threshold energy E have the same pair-production cross section for monopoles, limits for the cross section can be computed from the relation

$$N_m = L(E)N\sigma_{\gamma,\max}, \quad (2)$$

where for incoherent production (in the field of protons) $N = 2.7 \times 10^{23}$ is the number of protons (free or bound) per cm^3 of oil, and for coherent production (in the field of nuclei) $N = 3.4 \times 10^{22}$ equals the number of carbon atoms per cm^3 . The energy E is related to the monopole mass m by the equation

$$E = 2mc^2(1 + m/m_T), \quad (3)$$

where m_T is the mass of the target nucleus, or nucleon. For $mc^2 = 2.4$ BeV, for example, $E = 17$ BeV for incoherent, and $E = 5.8$ BeV for coherent production. For these energies, Fig. 3 gives $L = 6 \times 10^{10}$ cm and $L = 8 \times 10^{13}$ cm, respectively. Again setting $N_m = 2$, one obtains from Eq. (2) $\sigma_{\gamma,\max} = 1.3 \times 10^{-34}$ cm^2 per proton for the incoherent and $\sigma_{\gamma,\max} = 7 \times 10^{-37}$ cm^2 per nucleus for the coherent process. Because L rises steeply with decreasing E , $\sigma_{\gamma,\max}$ falls strongly as m decreases.

10. SENSITIVITY TO MONOPOLES OF "UNIT" CHARGE

In the foregoing discussion of the expected behavior of monopoles in this experiment, we have assumed a monopole charge $g = (137/2)e$. One has no theoretical grounds for excluding the possibility of a charge which is an integral multiple of $(137/2)e$. Indeed, some speculative arguments can be advanced favoring what we shall call the "unit" charge, $137e$. Examining the experimental conditions with this possibility in mind, we find a weak point at the step where monopoles made in the primary target are assumed to penetrate, in flight, the wall of the oil pot. Owing to the increased collision losses, in the upper mass range only those monopoles which emerge with nearly the maximum energy available will be able to penetrate the aluminum wall. Monopoles stopped in that wall remain exposed to the end field of the solenoid, which ranges in strength from 30 to 50 G over the region in question. From our earlier discussion (Sec. 2) of the migration of monopoles in crystals under the influence of a magnetic field, it seems possible that the monopole would eventually migrate into the liquid and thence be drawn over the normal route to the surface, to be extracted, accelerated, and recorded in the detector. But this is by no means certain. It was, of course, just the uncertainty of such considerations of migration and trapping that motivated our use of the liquid "catcher". Thus, to justify extending our conclusion to cover the case of the "unit"

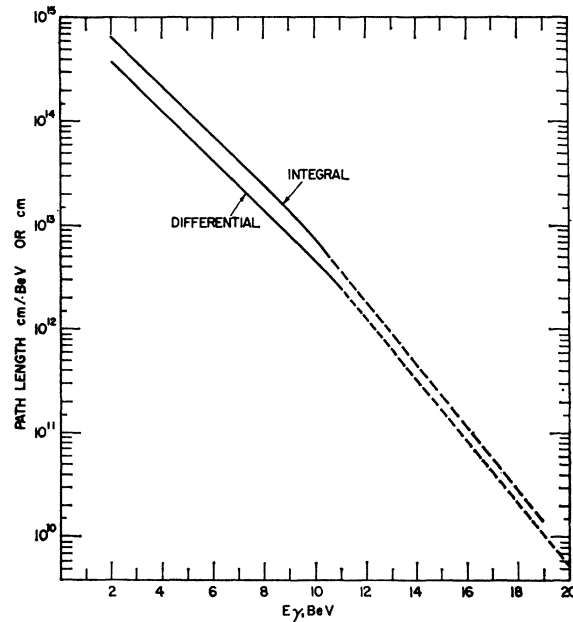


FIG. 3. Differential and integral spectra ($h\nu > E_\gamma$) of photon path length in the oil target for 2×10^{15} interacting protons. Dashed portions of the curves depend on the extrapolations in Fig. 2, but below 13–14 BeV little uncertainty is so introduced, because most of the contribution to the path length comes from photon emission angles for which extrapolation was not needed.

monopole, insofar as they may be produced in the primary target, a further test was required.

If we assume that monopoles are trapped in the aluminum wall so as to be immobilized against a field of 30 G over the duration of a run, then they will surely remain in place in the ambient field of 1 or 2 G that prevails when the solenoid is off. Such monopoles would simply accumulate in the apparatus, and could ultimately be extracted by the brief application of a very strong field. A final test of this sort was performed in a manner quite similar to the one used to extract monopoles possibly trapped at the oil surface (see Sec. 5). Again, the small coil was mounted in the large solenoid which was now closed at its bottom by a base plate. The part of the oil well facing the primary target—which had previously received secondaries from about 4×10^{16} circulating protons—was cut into disks. These disks were inserted, one after another, into the coil assembly at a height suitable for focusing. Then the large coil was energized to 500 G and the small one pulsed to about 10 kG. Monopoles of unit charge bound by less than about 10 eV should so have been extracted.

The response of the emulsion detector to unit-charge monopoles presents no difficulty. The energy acquired by such a monopole from the solenoid is doubled, as compared to that of the "half-charge" monopole, while the range is halved owing to the quadrupled collision loss. Making allowance for the energy loss in the windows and the short air path, one still finds that the track of such a monopole in the emulsion should have been

conspicuous under the conditions of scanning. Only if monopoles of unit charge were bound in aluminum by 0.02 to 0.2 eV and would thus have been drawn from the disks by a field between 30 and 300 G, prevailing while the field was rising, could their tracks have been so short, namely $\lesssim 50\mu$, as to escape detection. Although the scanners had been instructed to watch even for short tracks, the efficiency of finding them might have been low. Our earlier conclusions in regard to extraction from the liquid surface and stripping are not altered materially by the assumption of "unit" charge.

Two K0 emulsions were exposed and scanned, each having been used for 18 disks. No monopole-like track was found.

11. SUMMARY AND DISCUSSION

From our failure to observe monopoles, we have concluded that in collisions of 30-BeV protons with nucleons, where the highest monopole mass allowed by kinematics was 2.9 BeV, the upper limit for the production cross section σ_n is about 2×10^{-40} cm². Bradner and Isbell⁸ have derived a similar limit, 2×10^{-40} cm², from the negative result of an experiment in which a polyethylene target was bombarded by 6-BeV protons. At this energy poles lighter than the proton could have been made. They could have been found, in this experiment, if they were bound in the target material by 3–20 eV. Two experiments have been performed with proton energies similar to ours. Fidecaro *et al.*,⁴ using counters, found $\sigma_n \lesssim 2 \times 10^{-39}$ cm² for monopoles made, and bound, in aluminum or polyethylene targets. Amaldi *et al.*,⁵ using emulsions, found limits similar to, or somewhat higher than, ours in runs in which targets made of aluminum, polyethylene and aluminum, or a Cu-Cr alloy were first bombarded and then subjected to a strong magnetic field for extraction of poles bound with 0.6 to 60 eV. In another pair of experiments with a graphite target there was no restriction on the allowable binding energy, and the limit on σ_n was 10^{-39} cm².

Because the γ rays emerging from the primary target

have a continuous spectrum, no definite limit for the photoproduction cross section σ_γ can be given. Such limits would depend on the monopole mass as well as on the unknown dependence on γ -ray energy of σ_γ . As an example, we obtain for a monopole mass of 2.4 BeV and for the unrealistic case of σ_γ being independent of γ -ray energy above threshold limits of 1.3×10^{-34} cm² for production in the field of protons and 7×10^{-37} cm² for production in the field of carbon nuclei.

No limits for σ_γ are given in references 3–5. However, we estimate that in the search of Amaldi *et al.*,⁵ in which γ rays could interact in the target in which they were created, these limits should be roughly the same as ours, as in their experiment the acceptance angle is larger than ours and compensates for a lesser amount of matter traversed by a γ ray.

In our main experiment, about 10% of the circulating protons were used in the counter run. Although the limit set by the counters is, therefore, an order of magnitude higher than that of the emulsion, this detection technique provides us with comforting assurance that monopoles were not overlooked because of some unsuspected failure of the low-sensitivity emulsions to show monopole tracks. Only under two sets of conditions, both believed to be unlikely, would monopoles created in our experiment have been systematically missed. This would have occurred if the poles had become bound to nuclei at distances intermediate between nuclear and atomic dimensions, or if poles made in proton-nucleon collisions in the primary target had charge $137e$, a mass close to the kinematic limit, and were bound in aluminum by 0.02 to 0.2 eV.

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²² The numbers quoted here and below have been adjusted for the use of different confidence limits by multiplying the numbers of Bradner and Isbell and of Fidecaro *et al.* by 2 and those of Amaldi *et al.* by 0.7.